VECTOR

B.Sc. Part II

4th Paper

Some important points

1. If ϕ be a scalar point function then gradient of ϕ = grad ϕ =

$$\nabla\phi = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)\phi$$

$$= \vec{i}\frac{\partial\phi}{\partial x} + \vec{j}\frac{\partial\phi}{\partial y} + \vec{k}\frac{\partial\phi}{\partial z}$$ is

a vector point function.

2. <u>Divergence of a vector point function $\vec{f}$</u>

$$\text{div}\,\vec{f} = \nabla\cdot\vec{f} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)\cdot\vec{f}$$

$$= \vec{i}\cdot\frac{\partial\vec{f}}{\partial x} + \vec{j}\cdot\frac{\partial\vec{f}}{\partial y} + \vec{k}\cdot\frac{\partial\vec{f}}{\partial z}$$

and it is a scalar point function.

3. <u>Curl of a vector point function $\vec{f}$</u>

$$\text{curl}\,\vec{f} = \nabla\times\vec{f} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)\times\vec{f}$$

$$= \vec{i}\times\frac{\partial\vec{f}}{\partial x} + \vec{j}\times\frac{\partial\vec{f}}{\partial y} + \vec{k}\times\frac{\partial\vec{f}}{\partial z}$$

and curl $\vec{f}$ is a vector point function.

* If $\vec{f} = f_1\vec{i} + f_2\vec{j} + f_3\vec{k}$

then

$$\text{div } \vec{f} = \nabla\cdot\vec{f} = \vec{i}\cdot\frac{\partial \vec{f}}{\partial x} + \vec{j}\cdot\frac{\partial \vec{f}}{\partial y} + \vec{k}\cdot\frac{\partial \vec{f}}{\partial z}$$

$$\Rightarrow \text{div } \vec{f} = \vec{i}\cdot\frac{\partial}{\partial x}(f_1\vec{i} + f_2\vec{j} + f_3\vec{k}) + \vec{j}\cdot\frac{\partial}{\partial y}(f_1\vec{i} + f_2\vec{j} + f_3\vec{k})$$
$$+ \vec{k}\cdot\frac{\partial}{\partial z}(f_1\vec{i} + f_2\vec{j} + f_3\vec{k})$$

$$\Rightarrow \text{div } \vec{f} = \vec{i}\cdot\left(\frac{\partial f_1}{\partial x}\vec{i} + \frac{\partial f_2}{\partial x}\vec{j} + \frac{\partial f_3}{\partial x}\vec{k}\right)$$
$$+ \vec{j}\cdot\left(\frac{\partial f_1}{\partial y}\vec{i} + \frac{\partial f_2}{\partial y}\vec{j} + \frac{\partial f_3}{\partial y}\vec{k}\right)$$
$$+ \vec{k}\cdot\left(\frac{\partial f_1}{\partial z}\vec{i} + \frac{\partial f_2}{\partial z}\vec{j} + \frac{\partial f_3}{\partial z}\vec{k}\right)$$

Since $\vec{i}\cdot\vec{i} = 1 = \vec{j}\cdot\vec{j} = \vec{k}\cdot\vec{k} = 1$

and $\vec{i}\cdot\vec{j} = 0 = \vec{i}\cdot\vec{k} = \vec{j}\cdot\vec{i} = \vec{j}\cdot\vec{k} = 0,$

$\vec{k}\cdot\vec{i} = 0 = \vec{k}\cdot\vec{j}$

$$\Rightarrow \boxed{\text{div } \vec{f} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}}$$

where $\vec{f} = f_1\vec{i} + f_2\vec{j} + f_3\vec{k}$